VI-7. INTERPRETATION OF INTERACTION OF FABRY-PEROT RESONATOR FIELDS WITH PLASMAS

D. H. Auston, R. I. Primich, and R. A. Hayami

General Motors Defense Research Laboratories, Santa Barbara, California

Fabry-Perot, or open resonators are extremely useful in plasma diagnostics because of the excellent spatial resolution and sensitivity to small changes in electron density and collision frequency which can be achieved with them. Their application to diagnostics has been considered in detail (References 1 and 2).

In this earlier work the interaction of the resonator field with a plasma located in the focal region was considered by assuming that the field distribution was not perturbed by the presence of the plasma. In one particular method, the transmission coefficient of a planar resonator containing a plane slab of plasma was found by transmission line techniques. As an example, after approximations were made, the fractional shift in resonant frequency, due to the plasma slab, was found to be

$$\frac{\Delta f}{f_o} = -\frac{d\epsilon}{d_o} \Delta N,$$

where

$d\epsilon$ = slab thickness,

d_o = resonator length, and

ΔN = change in refractive index due to plasma density changes.

In this instance losses were neglected. Lonngren, et al, (Reference 3) have analyzed the same problem using, however, an incorrect application of the usual perturbation theory, i.e., Slater's Theorem (Reference 4). Their result for frequency shift is identical to the above and they extended the analysis to include plasma loss, accurate to the same order.

It has been found in practice that the approximate analyses referred to above are inadequate to explain actual observations, which is, no doubt, a consequence of the approximations made. For this reason a more thorough investigation has been undertaken of errors incurred in making these approximations. Preliminary results will be reported in this paper.

The rigorous transmission coefficient for a planar resonator containing a uniform slab of lossy plasma has been derived. This is so complicated that it is only useful in conjunction with extensive numerical calculations, which, however, can be carried out and do provide a direct check on approximate methods. As a first approximation, multiple reflections within the slab are neglected, which is equivalent to ignoring squares of ΔN and higher. The following results are obtained

$$\frac{\Delta f}{f_o} = -\frac{d\epsilon}{d_o} \cdot \Delta N \cdot \left(1 + \frac{\sin kd\epsilon}{kd\epsilon} \cdot \cos 2k X_o\right)$$

$$\frac{Q_o - Q}{Q_o Q} = \delta \left(1 + \frac{\sin kd\epsilon}{kd\epsilon} \cdot \cos 2k X_o\right)$$

where

$k = \frac{2\pi}{\lambda_o}$,

X_o = displacement of slab center from focal plane.

Q and Q_o = resonator Q's with and without the slab.

δ = measure of the slab loss ($N = N_o (1-j\delta)$).

It is seen that pronounced fluctuations are experienced as either the slab thickness or displacement is changed. The application of first order perturbation theory yields identical results.

For resonator and plasma configurations other than plane-parallel, it appears that perturbation techniques hold the most promise. The first order theory, however, must be used with caution since serious errors can be introduced by the following two effects when the plasma density is increased, namely:

1 Energy coupling between modes

2 Loss of energy due to scattering outside the resonator.

The first of these effects, mode coupling, has been studied quite extensively in the past (Reference 5) for closed resonant structures. In the present paper it is shown that these (closed system) techniques can be applied with reasonable accuracy to an open system if the coupling is restricted to take place between low loss modes only. The results show that a significant deviation from first order perturbation theory (Slater's Theorem) can occur when two low loss (high Q) modes are separated by a small frequency difference.

The loss of energy by scattering from a perturbing medium in a resonator is peculiar to open resonators and has not been studied previously. This mechanism, which we shall call scattering loss, has been analyzed by the authors to predict the change in Q due to scattering in a Fabry-Perot when an inhomogeneous plasma is present. The method of analysis is analogous to the Born approximation in scalar scattering theory, and depends on a knowledge of the scalar Green's Function of the open resonator. Since the resonant modes of the Fabry-Perot do not form a complete set of functions, some other set of basis functions must be used. In the example discussed in this paper, an open elliptic cylinder resonator is studied and the Green's Function is expressed in terms of the solutions of the scalar wave equation in elliptic cylinder coordinates. The derivation of the Green's Function is based on the summation of the multiple reflection from the Fabry-Perot reflectors of the free space Green's Function. The summation can be expressed as the inverse of a "reflection matrix". Once the Green's Function is found, it is a simple matter to calculate the radiated energy loss and thus the change in Q of the system.

Experimental studies have been performed with 5 gc and 35 gc Fabry-Perot resonators using low density polyfoam as the perturbing medium. The change in resonant frequency and Q have been measured for slabs, circular cylinders, and square cylinders. Good agreement is obtained with first order theory. Sample results are given in Figures 1 and 2. Further experiments with higher density materials are being performed at present to study mode coupling and scattering, and will also be reported.

ACKNOWLEDGMENT

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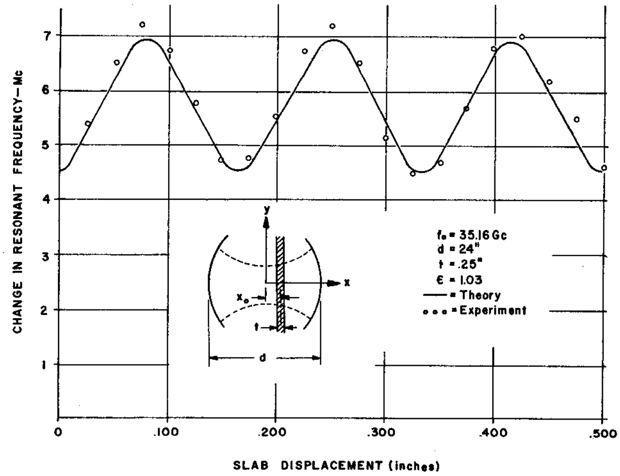

Figure 1. Frequency Shift of Slab as a Function of Slab Position

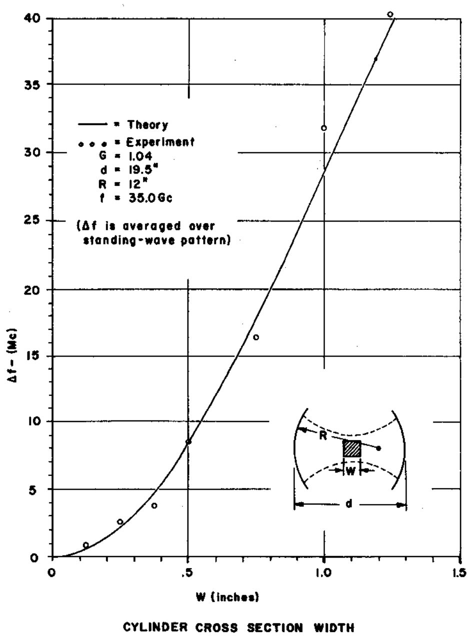

Figure 2. Perturbation by Square Cylinders

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